Spherical Symmetric Metric and Christoffel Symbols

General Spherically Symmetric Metric

$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2 \tag{1}$$

Metric Tensor and Inverse Metric

$$g_{\mu\nu} = \begin{pmatrix} -A(r) & 0 & 0 & 0 \\ 0 & B(r) & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}, \quad g^{\mu\nu} = \begin{pmatrix} -\frac{1}{A(r)} & 0 & 0 & 0 \\ 0 & \frac{1}{B(r)} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2\sin^2\theta} \end{pmatrix} \tag{2}$$

Christoffel Formula (Lower-Index)

$$\Gamma_{\mu\nu\lambda} = \tfrac{1}{2}\left(g_{\mu\nu,\lambda} + g_{\mu\lambda,\nu} - g_{\nu\lambda,\mu}\right) \tag{3}$$

Nonzero Lower-Index Christoffel Symbols

$$\begin{aligned}
&\Gamma_{001} = \Gamma_{010} = -\tfrac{1}{2}A'(r) \\
&\Gamma_{100} = \tfrac{1}{2}A'(r) \\
&\Gamma_{111} = \tfrac{1}{2}B'(r) \\
&\Gamma_{122} = \Gamma_{212} = \Gamma_{221} = -r \\
&\Gamma_{133} = \Gamma_{313} = \Gamma_{331} = -r\sin^2\theta \\
&\Gamma_{233} = -r^2\sin\theta\cos\theta \\
&\Gamma_{332} = \Gamma_{323} = r^2\sin\theta\cos\theta
\end{aligned}$$

Nonzero Raised-Index Christoffel Symbols

$$\Gamma^0_{01} = \Gamma^0_{10} = \frac{A'}{2A}$$
$$\Gamma^1_{00} = \frac{A'}{2B}$$
$$\Gamma^1_{11} = \frac{B'}{2B}$$
$$\Gamma^1_{22} = -\frac{r}{B}, \quad \Gamma^1_{33} = -\frac{r\sin^2\theta}{B}$$
$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r}$$
$$\Gamma^2_{33} = -\sin\theta\cos\theta$$
$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r}$$
$$\Gamma^3_{23} = \Gamma^3_{32} = \cot\theta$$

Antisymmetric Part of $\Gamma_{\mu\nu\lambda}$ in $\mu \leftrightarrow \nu$

$$\Gamma_{[\mu\nu]\lambda} = \tfrac{1}{2}(\Gamma_{\mu\nu\lambda} - \Gamma_{\nu\mu\lambda}) \tag{4}$$

$$\Gamma_{[01]0} = \tfrac{1}{2}(\Gamma_{010} - \Gamma_{100}) = \tfrac{1}{2}(-\tfrac{1}{2}A'(r) - \tfrac{1}{2}A'(r)) = -\tfrac{1}{2}A'(r)$$
$$\Gamma_{[01]1} = \tfrac{1}{2}(\Gamma_{011} - \Gamma_{101}) = 0$$
$$\Gamma_{[32]3} = \tfrac{1}{2}(\Gamma_{323} - \Gamma_{233}) = \tfrac{1}{2}(r^2\sin\theta\cos\theta + r^2\sin\theta\cos\theta) = r^2\sin\theta\cos\theta$$

Symmetric Part of $\Gamma_{\mu\nu\lambda}$ in $\mu \leftrightarrow \nu$

$$\Gamma_{(\mu\nu)\lambda} = \tfrac{1}{2}(\Gamma_{\mu\nu\lambda} + \Gamma_{\nu\mu\lambda}) \tag{5}$$

$$\Gamma_{(00)1} = \Gamma_{001} = -\tfrac{1}{2}A'(r)$$
$$\Gamma_{(11)1} = \Gamma_{111} = \tfrac{1}{2}B'(r)$$
$$\Gamma_{(12)2} = \Gamma_{122} = -r$$
$$\Gamma_{(13)3} = \Gamma_{133} = -r\sin^2\theta$$
$$\Gamma_{(23)3} = \tfrac{1}{2}(\Gamma_{233} + \Gamma_{323}) = 0$$

Nonzero Components of $L_{\mu\nu\gamma} = \frac{1}{2} g_{\mu\nu,\gamma}$

Component	Value
L_{001}	$-\frac{1}{2}A'(r)$
L_{111}	$\frac{1}{2}B'(r)$
L_{221}	r
L_{331}	$r\sin^2\theta$
L_{332}	$r^2\sin\theta\cos\theta$